

New forms of continuity (δ - α -continuous and contra δ - α -continuous)

Salima Mohamed Khader*

ABSTRACT

In this paper, we explore the aspects of new forms of continuity, by utilizing generalized open sets which play a significant role in General Topology and Real Analysis. Firstly, we start this paper by recalling the history and the developments of this subject, as well as we include the introduction of topology and the basic concepts in the topology, It defines types of open sets and continuity. In the second section of this paper we introduced and investigated a relatively new notion of open sets; namely δ - α -open sets. Additionally, the comparison of this new notion with α -open sets have been done. Finally, we apply the notion of δ - α -open sets in topological space to present and study a new classes of functions called δ - α -continuous and contra δ - α -continuous as a new generalization of contra continuity, These functions were compared with α -continuous and contra α -continuous.

1 GENERAL INTRODUCTION

Topology is an important and interesting area of mathematics.

The study will not only introduce new concepts and theorems but also put old ones, like continuous functions into new perspective. However, to say just this is to understate the significance of topology, it is so fundamental that its influence is evident in almost every other branch of mathematics. This makes the study of topology relevant to all who aspire to be mathematicians.

Generalized open sets are new and well-known and important notions in Topology and its applications. Many topologists are focusing their research on these topics and this amounted to many important and useful results. In this respect, the variously modified form of continuity by utilizing generalized open sets which play a significant role in General Topology and Real Analysis. Levine (1963) introduced the notion of semi-open sets. According to Cameron (2007), this notion was Levine's most important contribution to the topology field.

Njastad (1965) introduced and investigated a weak form of open sets called α -open sets (originally called α -sets) in topological space and since the advent of these notions, several research papers with interesting results in different aspects came to existence. A subset A of a topological space $(X; \tau)$ is

* قسم الرياضيات - كلية التربية - جامعة مصراتة.

called preopen if $A \subseteq Int(Cl(A))$. The term 'preopen' was used for the first time by Mashhour et al. (1982). Monsef et al. (1983) introduced the notions of β -open sets and β -continuity in topological space.

The notion of b -open sets or sp -open sets was introduced by Andrijević (1996) and Dontchev and Przemski (1996) respectively. This type of sets was discussed by Ekici and Caldas (2004) under the name of γ -open sets. The class of b -open sets generates the same topology as the class of preopen sets. Since the advent of these notions, several research paper with interesting results in different respects came to existence.

Raychaudhuri and Mukherjee (1993) introduced the notion of δ preopen sets and Park et al. (1997) introduced the notion of δ -semiopen sets.

Hdeib (1982), define the concept of W -closed subset of a space $(X; \tau)$ if contains all of its condensation points. In Al-zoubi and Al-nashef (2003) it was shown that the collection of all W -open subsets forms a topology that is finer than the original topology on X and many properties of that space were also discussed. It seems that the whole idea of Hdeib (1982) and Al-zoubi and Al-nashef (2003) came from the very well-known facts about the standard topology on the real line.

Continuity on topological space is an important basic subject in the study of General Topology and several branches of mathematics.

Dontchev (1996) introduced the notions of contra-continuity in topological spaces. He defined a function $f: X \rightarrow Y$ is contra continuous if the preimage of every open set of Y is closed in X . A new weaker form of this class of function introduced and investigated by Jafari and Noiri (2001, 2002) called contra α -continuous function.

The aim of this work is focused on some new classes of continuity and contra continuity via new type of open sets which is δ - α -open sets. The following figure is an enlargement of some previous well-known figures. Note that none of the implications is reversible.

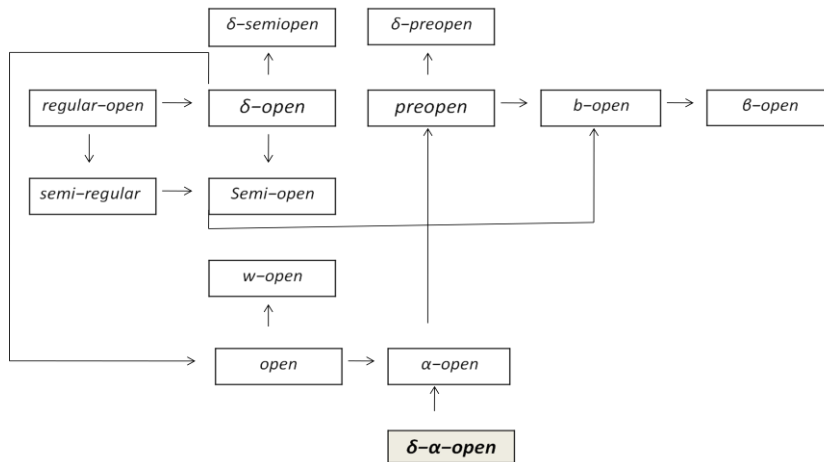


Figure 1 Relation among the sets in the topology τ

In this part we introduced general concepts in topology. After that, we shall state some concepts and results of the δ -open set and related topics that we need in the subsequent sections of this paper. These results have been given in the form of definitions, lemmas or theorems.

2 GENERAL CONCEPTS IN TOPOLOGY

Definition 2.1 [1] Let X be a set. A topology τ on X is a collection of subsets of X , each called an open set, such that

1. \emptyset and X are open sets;
2. The intersection of finitely many open sets is an open set;
3. The union of any collection of open sets is an open set.

The set X together with a topology τ on X is called topological space, and it is denoted by (X, τ) .

Throughout the present paper, the space X and Y or $((X, \tau)$ and (Y, σ)) always mean topological spaces.

Definition 2.2 [1] A subset A of a topological space X is closed if the set $X - A$ is open.

Definition 2.3 [1] Let A be a subset of a topological space X . The interior of A , denoted $Int(A)$, is the union of all open sets contained in A . The closure of A , denoted $Cl(A)$, is the intersection of all closed sets containing A . For a subset A of a space X , the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively.

Definition 2.4 [1] Let X and Y be topological spaces. A function $f: X \rightarrow Y$ is continuous if $f^{-1}(A)$ is open in X for every open set A in Y . We call this the open set definition of continuity.

3 BASIC CONCEPTS OF α -OPEN SET

Let (X, τ) be a space and A is a subset of X . A subset A of a space X is said to be regular open (respectively regular closed) (Stone 1937) if $A = \text{Int}(Cl(A))$ (respectively $A = Cl(\text{Int}(A))$). Veličko (1968) introduced δ -interior of a subset A of X as the union of all regular open sets of X contained in A and is denoted by $\delta\text{-Int}(A)$. The subset A is called δ -open if $A = \delta\text{-Int}(A)$, i.e, a set is δ -open if it is the union of regular open sets. The complement of a δ -open set is called δ -closed. The δ -closure of a set A in a space (X, τ) is defined by: $\{x \in X : A \cap \text{Int}(Cl(B)) \neq \emptyset, B \in \tau \text{ and } x \in B\}$ and it is denoted by $\delta\text{-Cl}(A)$. The concepts of δ -open sets are a stronger notion of open set.

Definition 3.1 A subset A of a space (X, τ) is said to be:

1. [11] semi-open if $A \subseteq Cl(\text{Int}(A))$ and semi-closed if $\text{Int}(Cl(A)) \subseteq A$;
2. [15] α -open if $A \subseteq \text{Int}(Cl(\text{Int}(A)))$ and α -closed if $Cl(\text{Int}(Cl(A))) \subseteq A$;
3. [12] preopen if $A \subseteq \text{Int}(Cl(A))$ and preclosed if $Cl(\text{Int}(A)) \subseteq A$;
4. [14] β -open or [3] semi-preopen if $A \subseteq Cl(\text{Int}(Cl(A)))$ and β -closed if $\text{Int}(Cl(\text{Int}(A))) \subseteq A$;
5. [18] δ -preopen if $A \subset \text{Int}(\delta\text{-Cl}(A))$ and δ -preclosed if $Cl(\delta\text{-Int}(A)) \subset A$;
6. [4] b -open or [7] sp -open sets if $A \subseteq Cl(\text{Int}(A)) \cup \text{Int}(Cl(A))$;
7. [17] δ -semiopen if $A \subset Cl(\delta\text{-Int}(A))$ and δ -semiclosed if $\text{Int}(\delta\text{-Cl}(A)) \subset A$;

Let (X, τ) be a space and A a subset of X : A point x is a limit point of a set A if every open set containing x contains at least one point of A distinct from x . Particular kinds of limit point are W -accumulation point, for which every open set containing x must contain infinitely many points of A , and condensation points, for which every open set containing x must contain uncountably many points of A . A set A is said to be W -closed (Hdeib 1982) if it contains all its condensation points. The complement of an W -closed set is said to be W -open. It is well known that a subset W of a space (X, τ) is W -open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U - W$ is countable. We note that the collection of all α -open subset of X is a topology on X , called the δ -topology, which is finer than the original one. A set $A \subseteq X$ is α -open if and only if A is semi-open and preopen set. Some authors use the term γ -open set for b -open set.

Definition 3.2 [19] Let A be a subset of a space X .

1. The α -closure of A , denoted by $\alpha\text{-Cl}(A)$, is the smallest α -closed set containing A . It is well-known that $\alpha\text{-Cl}(A) = A \cup Cl(\text{Int}(Cl(A)))$.
2. The α -interior of A , denoted by $\alpha\text{-Int}(A)$, is the largest α -open set contained in A .

4 PROPERTIES OF α -OPEN SET

In order to prove our results and the benefit of the reader we recall some basic well known results.

Theorem 4.1 [1] Let X be a space and A and B be subsets of X .

1. If U is an open set in X and $U \subset A$, then $U \subset Int(A)$.
2. If C is a closed set in X and $A \subset C$, then $Cl(A) \subset C$.
3. If $A \subset B$ then $Int(A) \subset Int(B)$.
4. If $A \subset B$ then $Cl(A) \subset Cl(B)$.
5. If A is open if and only if $A = Int(A)$.
6. A is closed if and only if $A = Cl(A)$.

Example 4.1 Let $A = \{0, 1\}$ and $B = \{0, 1, 2\}$. Then: $Int(A) = \{0, 1\} \subset Int(B) = \{0, 1, 2\}$ since $A \subset B$. Thus: $Cl(A) = \{0, 1\} \subset Cl(B) = \{0, 1, 2\}$ since $A \subset B$.

5 FUNCTIONS OF α -OPENSET

In this section, we recall some known notions, functions, and results which will be used in the work.

Definition 5.1 [13] A function $f: X \rightarrow Y$ is said to be α -continuous if $f^{-1}(V)$ is α -open in X for each open set V of Y .

Definition 5.2 A function $f: X \rightarrow Y$ and V be subset of Y is said to be:

1. [7] contra-continuous if $f^{-1}(V)$ is closed in X for each open set of Y .
2. [9] contra α -continuous if $f^{-1}(V)$ is α -closed in X for each open set of Y .

6 δ - α -OPEN SETS

We introduce the following relatively new definition.

Definition 6.1 A subset A of a space X is called:

1. δ - α -open if $A \subseteq Int(Cl(\delta-Int(A)))$.
2. δ - α -closed if $Cl(Int(\delta-Cl(A))) \subseteq A$.

Based on Definition 6.1 we now illustrate the following example:

Example 6.1 Let $X = \{x, y, w, z\}$ and let $\tau = \{\phi, X, \{x\}, \{w\}, \{x, y\}, \{x, w\}, \{x, y, w\}, \{x, w, z\}\}$. Then:

1. the set $\{w\}$ is δ - α -open since $Int(Cl(\delta-Int(\{w\}))) = \{w\}$. Thus $\{w\}$ is δ - α -open.
2. the set $\{x, z\}$ is not δ - α -open because $Int(Cl(\delta-Int(\{x, z\}))) = \phi$. Thus $\{x, z\}$ is not δ - α -open.

Theorem 6.2 Every δ - α -open is α -open.

Proof. Let A is δ - α -open. Then we have $A \subseteq Int(Cl(\delta-Int(A)))$. Claim, $\delta-Int(A) \subseteq Int(A)$. Let $x \in \delta-Int(A)$. So there exists δ -open set; namely U such that $x \in U \subseteq A$.

Since every δ -open is open. Then $x \in \text{Int}(A)$ and thus $\delta\text{-Int}(A) \subseteq \text{Int}(A)$ It follows that $\text{Cl}(\delta\text{-Int}(A)) \subseteq \text{Cl}(\text{Int}(A))$ and thus $\text{Int}(\text{Cl}(\delta\text{-Int}(A))) \subseteq \text{Int}(\text{Cl}(\text{Int}(A)))$.

We obtain $A \subseteq \text{Int}(\text{Cl}(\text{Int}(A)))$. Hence A is α -open.

The following example shows that theorem 6.3 is not reversible.

Example 6.2 Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{b, c, d\}, \{b\}, \{a, b\}\}$. Then $\{c, b\}$ is α -open since $\text{Int}(\text{Cl}(\text{Int}(\{c, b\}))) = X$. Thus $\{c, b\}$ is, α -open but it is not δ - α -open because $\text{Int}(\text{Cl}(\delta\text{-Int}(\{c, b\}))) = \phi$. Thus $\{c, b\}$ is not δ - α -open.

Theorem 6.3 Every α -closed is δ - α -closed.

Proof. Let A is α -closed set. Then we have $\text{Cl}(\text{Int}(\text{Cl}(A))) \subseteq A$. Claim, $\text{Cl}(A) \subset \delta\text{-Cl}(A)$. Let $x \in \text{Cl}(A)$. For all U be open set we have $A \cap U \neq \emptyset$, but $U \subset \text{Cl}(U)$ and $\text{Int}(U) \subset \text{Cl}(\text{Int}(U))$. Since U is open and $U \subset \text{Int}(\text{Cl}(U))$. Then $A \cap U \subset A \cap \text{Int}(\text{Cl}(U))$. But $A \cap U \neq \emptyset$, then $A \cap \text{Int}(\text{Cl}(U)) \neq \emptyset$. Thus, $x \in \delta\text{-Cl}(A)$. It follows that $\text{Int}(\text{Cl}(A)) \subseteq \text{Int}(\delta\text{-Cl}(A))$ and thus $\text{Cl}(\text{Int}(\text{Cl}(A))) \subseteq \text{Cl}(\text{Int}(\delta\text{-Cl}(A)))$. We obtain $\text{Cl}(\text{Int}(\delta\text{-Cl}(A))) \subseteq A$. Hence A is δ - α -closed.

Example 6.3 Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{b, c, d\}, \{b\}, \{a, b\}\}$. Then $\{b, c, d\}$ is α -closed since $\text{Cl}(\text{Int}(\text{Cl}(\{b, c, d\}))) \subseteq \{b, c, d\}$ and also is δ - α -closed since $\text{Cl}(\text{Int}(\delta\text{-Cl}(\{b, c, d\}))) \subseteq \{b, c, d\}$.

Definition 6.4 Let A be a subset of a space X .

1. The intersection of all δ - α closed sets containing A is called the δ - α -closure of A and is denoted by δ - α - $\text{Cl}(A)$.
2. The δ - α -interior of A is defined by the union of all δ - α -open sets contained in A and is denoted by δ - α - $\text{Int}(A)$.

Proposition 6.5 Let A be a subset of a topological space X . A point $x \in \delta$ - α - $\text{Int}(A)$ is called an δ - α -interior point of A if x belongs to δ - α -open set U contained in A : $x \in U \subset A$ where U is δ - α -open. The set of δ - α -interior point of A , denoted by

δ - α - $\text{Int}(A)$ is called the δ - α -interior of A .

Theorem 6.6 Let A and B be subsets of a space X . The following statements hold :

1. If $A \subset B$ then δ - α - $\text{Cl}(A) \subset \delta$ - α - $\text{Cl}(B)$;
2. If $A \subset B$ then δ - α - $\text{Int}(A) \subset \delta$ - α - $\text{Int}(B)$;
3. δ - α - $\text{Int}(A) \subset \alpha$ - $\text{Int}(A) \subset \alpha$ - $\text{Cl}(A) \subset \delta$ - α - $\text{Cl}(A)$;
4. $X \setminus \delta$ - α - $\text{Int}(A) = \delta$ - α - $\text{Cl}(X \setminus A)$;
5. δ - α - $\text{Int}(X \setminus A) = X \setminus \delta$ - α - $\text{Cl}(A)$.

Proof. 1. Let $A \subset B$. Our claim to show that δ - $\text{Cl}(A) \subset \delta$ - $\text{Cl}(B)$. Let $x \in \delta$ - $\text{Cl}(A)$. So any U open set in τ containing x . We have $A \cap \text{Int}(\text{Cl}(U)) \neq \emptyset$.

But $A \subset B$. Then $A \cap \text{Int}(Cl(U)) \subset B \cap \text{Int}(Cl(U))$. It follows that $x \in \delta - Cl(B)$ and thus $\text{Int}(\delta - Cl(A)) \subset \text{Int}(\delta - Cl(B))$ and then $Cl(\text{Int}(\delta - Cl(A))) \subset Cl(\text{Int}(\delta - Cl(B)))$.

2. Let $x \in \delta - \alpha - \text{Int}(A)$. Then there exists $\delta - \alpha$ -open set; namely U such that $x \in U \subseteq A$, but $A \subset B$. We obtain $x \in U \subseteq B$. Thus, $x \in \delta - \alpha - \text{Int}(B)$.

3. First we prove $\delta - \alpha - \text{Int}(A) \subset \alpha - \text{Int}(A)$. Let $x \in \delta - \alpha - \text{Int}(A)$. Then there exists $\delta - \alpha$ -open set; namely U such that $x \in U \subseteq A$. By theorem 6.3. U is α -open set. Hence $x \in \alpha - \text{Int}(A)$. Next, we prove $\alpha - \text{Int}(A) \subset \alpha - Cl(A)$. Since $\alpha - \text{Int}(A)$ is the largest α -open set contained in A , i.e. $\alpha - \text{Int}(A) \subseteq A$ and since $\alpha - Cl(A)$ is the smallest α -closed set containing A , i.e. $A \subseteq \alpha - Cl(A)$. Hence $\alpha - \text{Int}(A) \subset \alpha - Cl(A)$. Finally, we prove $\alpha - Cl(A) \subset \delta - \alpha - Cl(A)$. From theorem 6.5 we have $Cl(A) \subset \delta - Cl(A)$. Now let $x \in \alpha - Cl(A)$ so $x \in Cl(\text{Int}(Cl(A)))$. Then $x \in Cl(\text{Int}(\delta - Cl(A)))$. Thus, $x \in \delta - \alpha - Cl(A)$.

4. Let $x \in (X \setminus \delta - \alpha - \text{Int}(A))$. So for all U be α -open set containing X , we have $(X \setminus A) \cap U \neq \emptyset$. Thus, $x \in \delta - \alpha - Cl(X \setminus A)$.

5. Let $x \in \delta - \alpha - \text{Int}(X \setminus A)$. So there exists U be $\delta - \alpha$ -open set such that $U \subset (X \setminus A)$. Then we obtain $U \cap A = \emptyset$. It follows that $x \notin \delta - \alpha - Cl(A)$. Thus, $x \in (X \setminus \delta - \alpha - Cl(A))$.

Example 6.6 Let $X = \{x, y, w, z\}$, let $\tau = \{\emptyset, X, \{x\}, \{w\}, \{x, y\}, \{x, w\}, \{x, y, w\}, \{x, w, z\}\}$, let $A = \{x, w\}$ and $B = \{x, y, w\}$. Then: Let $w \in \delta - \alpha - \text{Int}(A)$. Then there exists $\delta - \alpha$ -open set: namely $\{w\}$ such that $w \in \{w\} \subseteq A$, but $A \subset B$. We obtain $w \in \{w\} \subseteq B$. Thus $w \in \delta - \alpha - \text{Int}(B)$. So, $\delta - \alpha - \text{Int}(A) \subset \delta - \alpha - \text{Int}(B)$.

7 $\delta - \alpha$ -CONTINUOUS AND CONTRA $\delta - \alpha$ -CONTINUOUS

The notions of α -continuous, contra-continuous and contra α -continuous were introduced and investigated by Mashhour et al. (1983), Dontchev (1996) and Jafari and Noiri (2001) respectively. In this section, we apply the notion of $\delta - \alpha$ -open sets in topological space to present and study a new class of functions called $\delta - \alpha$ -continuous and contra $\delta - \alpha$ -continuous functions.

7.1 $\delta - \alpha$ -Continuous

A function $f : X \rightarrow Y$ is said to be [13] α -continuous if $f^{-1}(V)$ is α -open in X for each open set V of Y . In this section, we introduce a new type of continuity called $\delta - \alpha$ -continuous.

Definition 7.1.1 A function $f : X \rightarrow Y$ is said to be $\delta - \alpha$ -continuous if $f^{-1}(A)$ is $\delta - \alpha$ -open in X for every open set A of Y .

Theorem 7.1.2 Every $\delta - \alpha$ -continuous function is α -continuous function.

Proof. Let $f: X \rightarrow Y$ be δ - α -continuous. Let V be open set in Y . Then $f^{-1}(V)$ is δ - α -open in X . By theorem 6.3 we obtain $f^{-1}(V)$ is α -open in X . Hence f is α -continuous.

The converse implications do not hold as it is show in the following example.

Example 7.1.1 Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{b, c, d\}, \{b\}, \{a, b\}\}$. Define a function $f: X \rightarrow X$ such that $f(a) = c, f(b) = b, f(c) = a$ and $f(d) = d$ then f is α -continuous but it is not δ - α -continuous since $f^{-1}(\{a, b\}) = \{c, b\}$ which is α -open not δ - α -open.

7.2 Contra δ - α -Continuous

We begin by recalling the following definitions. Next, we introduce a relatively new notion.

Definition 7.2.1 A function $f: X \rightarrow Y$ is said to be:

1. [6] contra-continuous if $f^{-1}(V)$ is closed in X for each open set of Y .
2. [9] contra α -continuous if $f^{-1}(V)$ is α -closed in X for each open set of Y .

Definition 7.2.2 A function $f: X \rightarrow Y$ is called contra δ - α -continuous if $f^{-1}(A)$ is δ - α -closed in X for every open set A of Y .

Theorem 7.2.3 Every contra α -continuous function is contra δ - α -continuous function.

Proof. Let $f: X \rightarrow Y$ be contra α -continuous. Let V be open set in Y . Then $f^{-1}(V)$ is α -closed in X . By theorem 6.5 we obtain $f^{-1}(V)$ is δ - α -closed in X . Hence f is contra δ - α -continuous.

Example 7.2.1 Let $X = Y = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then the function $f: (X, \tau) \rightarrow (X, \tau)$ defined as: $f(a) = b, f(b) = a$ and $f(c) = a$, is neither contra α -continuous nor contra δ - α -continuous since $f^{-1}(\{b\}) = \{a\}$ which is neither α -closed nor δ - α -closed.

Example 7.2.2 Let $X = Y = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b, c\}, \{a, b, c\}\}$. Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ as follows: $f(a) = d, f(b) = b, f(c) = b, f(d) = c$. Then f is contra α -continuous and contra δ - α -continuous since $f^{-1}(\{b, c\}) = \{b, c, d\}$ which is α -closed and δ - α -closed.

CONCLUSION

Some types of sets play an essential role in the study of various properties in topological spaces. In addition to this, the importance of continuity is significant in various areas of mathematics and related sciences. In this paper, we have proposed relatively new notions of open sets; namely δ - α -open set, this has been compared with α -open set using examples and theorems, as well as introducing new classes of functions called δ - α -continuous and

contra δ - α - continuous. These functions were compared with α - continuous and contra α - continuous. The comparison resulted in examples and theorems.

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